Should We Define Forecast Error as e = F – A *or* e = A – F?

Survey edited by Kesten Green and Len Tashman

THE ISSUE

Suppose we forecast sales of 900 units for the month just gone (F), and actual sales (A) were recorded as 827 units. The difference between the two figures, 73 units, is the magnitude of the forecast error.

One way to express the forecast error is as A minus F (A - F), which yields an error of:

e = 827 - 900 = -73 units.

Alternatively, the forecast error can also be expressed as F minus A (F - A). Using this formulation, the forecast error for the month just gone is:

e = 900 – 827 = +73 units.

Does it matter which formula we use?

THE SURVEY

In March of this year, Kesten Green sent the following message to the membership of the International Institute of Forecasters (IIF):

Dear All,

How should "forecast error" be defined? A Google Scholar search for "error 'forecast minus actual'" and for the reverse formulation turn up 51 and 46 hits respectively – almost a tie.

I asked a small sample of senior IIF members (two) whether they prefer to define forecast error as A - F or as F - A, and why. Again, opinion was divided, with one preferring the A - F definition as derived from the basic statistical model formulation of A = F + e, while the other preferred the more intuitively appealing F - A whereby a positive error means that the forecast was too high.

I'd like to know what you think: Which do you prefer, and why?

Regards, Kesten

By the time of writing, eleven responses had been received, with more than half preferring to calculate error as A - F.

Respondents who preferred F - A all reasoned that it was more intuitive that a positive error represented an over-forecast and a negative error an under-forecast. F - A is also more consistent with concepts of bias.

Respondents who preferred the A - F formulation argued that statistical convention, ease of statistical calculation, investment in software that adhered to statistical convention, and plain pragmatism provided justification. Two fans of A - F also suggested that this version is intuitive when assessing performance against a budget or plan, because a positive value indicates that a budget has been exceeded or a plan has been surpassed.

Here is an edited sampling of the individual responses:

SUPPORT FOR A – F

1. Can't say I've ever thought of "why," since the statistical measure is always A - F, with the basic idea being:

A = forecast model + forecast error.

This basic concept provides the justification. Obviously there is no mathematical reason why it could not be forecast model minus forecast error, but that would be more complex and therefore not sensible.

2. I use Actual minus Forecast. I am a pragmatist and do not believe there is a right answer, merely a need to settle the answer by convention. I am saying that there is, as a matter of fact, no basis for finding a right answer, and seeking one is fruitless; thus the need for a convention. Of course, all of us will be attached to the method we frequently use and will easily find justifications for its correctness.

3. In statistical terms, the forecast is an expected value. A deviation in statistical computations is actual minus mean or other expected value. Thus, error = A - F is consistent with standard statistical calculations, actual minus mean.

In planning and control settings, the sign of the deviation can be important in the context of a negative feedback control loop.

There are other explanations; however, none preclude the opposite definition, but an additional operation (subtraction) would be necessary to make e = F - A operable in the planning and control settings.

4. In seismology, where the sign of prediction error does matter, (model) forecasted travel time comes with "–," i.e., we use error = A - F. Effectively, when actual seismic wave arrives before the time predicted by model we have negative travel time residual (error).

5. I agree that A-F is counterintuitive in that a + error means that a forecast was too low.

However, A - F makes sense for people using forecasts to set budgets or make plans (e.g. a + value would show that the budget or plan has been exceeded).

Exponential smoothing corrects for its past errors. In its specification A - F arguably makes life a bit simpler as we have Ft+1 = Ft + alpha * error, rather than Ft+1 = Ft - alpha * error, which may be a bit more difficult to explain.

In regression, fitted residuals and forecast errors are measured in the same way if we stick to A - F. If we were also to start using F - A for residuals, then the whole of regression analysis and its associated software would need to be revised.

6. I use A - F, and I do feel this is the mathematically correct answer. [Respondent #5] has already provided an excellent defense of this position, so I won't bother to elaborate further.

But since both formulae appear to be commonly used, how does a forecaster communicate a signed forecast error to someone else?

Obviously, if both parties know the convention being used, then there is not an issue. Unfortunately, I feel that even if the participants in this discussion were to decide upon a convention, it is unlikely to be universally

adopted in the near future. So what's a forecaster to do when confronted with having to communicate a signed forecast error to someone who is ignoring this thread? I would suggest that we teach our students to use the words "over-forecasted" and "under-forecasted."

Stating "I over-forecasted by 200 units" is unambiguous, and conveys the same information as "my forecast error was -200" (to an A - F type like me).

7. I think that A - F makes a lot more sense; as in common financial terms, F would be budget (B) and a positive Actual minus Budget would be over budget while a negative would be under budget.

SUPPORT FOR F – A

1. The one advantage of F - A is that it fits intuition that a positive error is an over-forecast and a negative error an under-forecast.

2. I would prefer and use forecast error as F - A since it is easy to explain that positive bias means forecast is higher than actual and negative bias means forecast is lower than actual.

3. It makes intuitive sense to express error in the same way that bias is interpreted, i.e. F - A, where positive indicates "over."

More importantly: From a business point of view, it only really matters that you track it against history as a relative point of reference to gauge improvement (or not). So it's really up to the users.

Most error (not bias) measures discard the sign in favor of working with absolute or standardized data, so the effect is minimal there.

4. When I'm in a training session and one of the students says, "My error was too high (or too big or too positive)," this means that it was the forecast that was too high; forecast bigger than the observed value means that the error in popular vernacular is positive. If, on the other hand, he says his error was very negative, then what he means to say is that the forecast was lower than the observed value. In common vernacular, the reference point is the observed value and the forecast is compared to it, either too high or too low, either too positive or too negative.

Mathematically, it's better (or easier or more consistent) to use A = F + e for the few in the human race who believe that mathematical propositions are more reasonable (or scientific, or structured). To understand what this means – that F = A + e doesn't work very well for mathematical formulations – I had to go to graduate school.

NEXT ISSUE

Perhaps a more substantive issue than A - F vs. F - A arises when we wish to report a percentage error. The question is what to include in the denominator: the actual A, the forecast F, an average of A and F, or something else. Tell us what you think by contacting:

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